FREE CONVECTIVE HEAT EXCHANGE OF A CYLINDER WITH AN EXPONENTIALLY DECAYING HEAT FLUX ON THE SURFACE

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Results of a numerical calculation of heat liberation from a vertical cylinder to the surrounding medium are presented. The data obtained are generalized to a wide range of problem parameters.

Heat exchange of a vertical surface under free convection conditions is determined by the form and orientation of the surface relative to the mass force vector, the boundary conditions, and the physical properties of the surrounding fluid. The literature offers studies which theoretically and experimentally investigate heat liberation from a vertical cylinder with constant temperature and thermal flux on the surface [1, 2]. Recently, there has arisen in the development of high-temperature radio apparatus the necessity of calculating heat liberation from vertical cylindrical surfaces, various portions of which are heated nonuniformly. If the thermal source is located at the bottom of the body, then the decrease in heat flux in the upper portion may be approximated by an exponential dependence

$$q_w = q_0 \exp(-x/m). \tag{1}$$

We will consider free motion of a liquid about a vertical cylinder heated at its base. We assume that the liquid is incompressible, that its physical properties are constant, and that viscous dissipation may be neglected. In the Boussinesq approximation the equations of motion and heat exchange and the boundary conditions may be written in the form

$$\frac{\partial ur}{\partial x} + \frac{\partial vr}{\partial r} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + g\beta\vartheta, \tag{3}$$

$$u\frac{\partial \vartheta}{\partial x} + v\frac{\partial \vartheta}{\partial r} = \frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \vartheta}{\partial r}\right), \tag{4}$$

$$u = v = 0,\ -\lambda\frac{\partial \vartheta}{\partial r} = q_0 \exp\left(-\frac{x}{m}\right) \quad \text{as} \quad r = R,$$

$$u = 0,\ \vartheta = 0 \quad \text{as} \quad r \to \infty. \tag{5}$$

We introduce the flow function $\psi(x, r)$ from continuity equation (2) and introduce the following variables to the problem of Eqs. (2)-(5)

$$\psi = 5\nu R\left(\frac{\mathrm{Gr}_x^*}{5}\right)^{1/5} f(\xi,\ \eta),\ \xi = \frac{2x}{R}\left(\frac{\mathrm{Gr}_x^*}{5}\right)^{-1/5},$$

$$\vartheta = \theta(\xi,\ \eta)\frac{q_0 x}{\lambda}\left(\frac{\mathrm{Gr}_x^*}{5}\right)^{-1/5},\ \eta = \left(\frac{\mathrm{Gr}_x^*}{5}\right)^{1/5}\frac{r^2 - R^2}{2xR}. \tag{6}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 3, pp. 508-512, September, 1979. Original article submitted February 9, 1979.

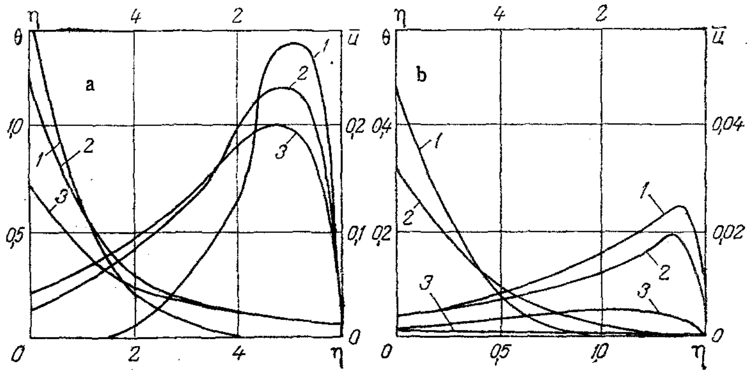

Fig. 1. Dimensionless velocity and temperature in boundary layer: a) Pr = 0.7; 1) $\xi = 0$; 2) $\xi = 1$, $\xi_m^5 = 20$; 3) $\xi = 1$, $\xi_m = 1$; b) Pr = 100; 1) $\xi = 0$; 2) $\xi = 2$, $\xi_m^5 = 10$; 3) $\xi = 2$, $\xi_m = 1$.

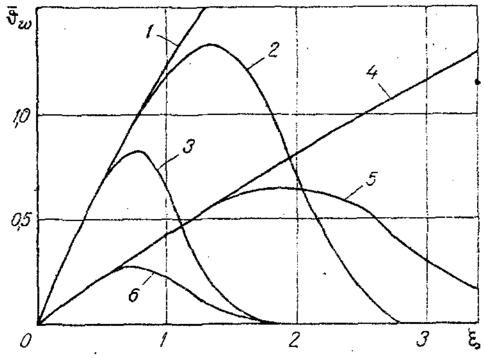

Fig. 2. Dimensionless excess temperature versus cylinder height: Pr = 0.7: 1) $\xi_m^5 = \infty$; 2) 20; 3) 1; Pr = 100; 4) $\xi_m^5 = \infty$; 5) 20; 6) 1.

Equations (3), (4) and boundary condition (5) are now rewritten in the form

$$(1+\xi\eta)\frac{\partial^3 f}{\partial\eta^3}+4f\frac{\partial^2 f}{\partial\eta^2}-3f\left(\frac{\partial f}{\partial\eta}\right)^2+\xi\frac{\partial^2 f}{\partial\eta^2}+\theta=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial\xi}-\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial\xi}\right), \qquad (7)$$

$$\frac{1}{\mathrm{Pr}}(1+\xi\eta)\frac{\partial^2\theta}{\partial\eta^2}+4f\frac{\partial\theta}{\partial\eta}-\theta\frac{\partial f}{\partial\eta}+\frac{1}{\mathrm{Pr}}\xi\frac{\partial\theta}{\partial\eta}=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right), \qquad (8)$$

$$\frac{\partial f}{\partial\eta}=f=0,\quad \frac{\partial\theta}{\partial\eta}=-\exp(-\xi^5/\xi_m^5)\ \text{as}\ \eta=0,$$

$$\frac{\partial f}{\partial\eta}=0,\quad \theta=0\ \text{as}\ \eta=\infty. \qquad (9)$$

The varying heat flux on the wall introduces an additional nonisothermal parameter $\xi_m = 2m/R(\mathrm{Gr}_m^*/5)^{-1/5}$, which represents the dimensionless longitudinal coordinate ξ at the point x = m.

We note the characteristic peculiarities of the problem of Eqs. (7)-(9). At $\xi = 0$ it transforms to a self-similar problem for a plane wall at q_w = const, while as $\xi_m \to \infty$ the equations describe free convective heat exchange of a vertical cylinder with q_w = const on the surface [2].

Equations (7), (8) with boundary conditions (9) were calculated for Pr = 0.7-100 and ξ = 0-5. The numerical calculations were carried out by an implicit difference scheme with six nodes. Derivatives were approximated by finite differences to a second-order accuracy in the parameters ξ and η [3].

The accuracy of the calculations was monitored by performing identical calculations with steps $\Delta\xi$ and $\Delta\eta$ twice as small, and an accuracy of 1-1.5% was shown in all cases.

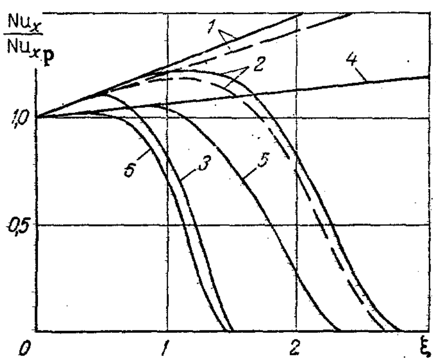

Fig. 3

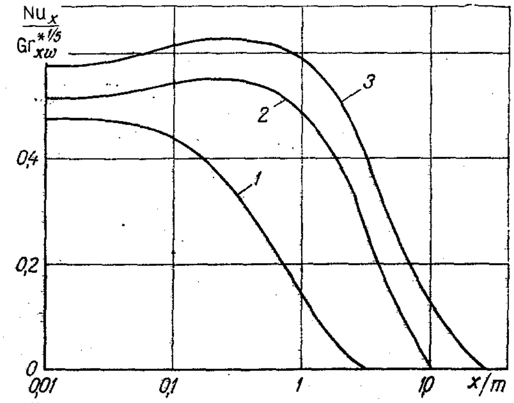

Fig. 4

Fig. 3. Relative cylinder heat liberation coefficient versus height Nu_x/Nu_{xp} (for notation see Fig. 2). Dashed lines show calculation by thin layer method with consideration of surface curvature.

Fig. 4. Effect of curvature on local heat liberation coefficient distribution over height (Pr = 0.7): 1) $\xi_m^5 = 0$; 2) 1; 3) 20.

The distribution of dimensionless velocity $\bar{u} = [u/5(\nu/x)](Gr_x^*/5)^{2/5}$ and temperature θ in the boundary layer is shown in Fig. 1a, b. The decrease in wall temperature with height leads to a sharp reduction in flow. This is shown in the drop in velocity and temperature with increase in the parameter ξ_m. At Pr = 100 the decrease in dynamic boundary layer and increase in thickness of the thermal boundary layer are more significant.

The dimensionless excess wall temperature decreases with longitudinal coordinate ξ. However, the change in excess wall temperature is of a different character, since the longitudinal coordinate ξ appears in the scale θ_W. Therefore, Fig. 2 shows the excess wall temperature relative to the constant scale.

$$\bar{\vartheta}_w = \vartheta_w \Big/ \frac{q_0 R}{2\lambda} = \theta_w \xi.$$

At the base the wall temperature is equal to that of the surrounding medium, and in the initial portion the wall temperature increases as in the case q_W = const. For large ξ the decrease in thermal flux with longitudinal coordinate causes a reduction in wall temperature and equalization to the temperature of the surrounding medium. The wall temperature maximum increases with increase in ξ_m and reduction in Pr. The value of the temperature maximum as well as its location are significant in calculating the degree to which electronic devices are heated. Also of interest is the characteristic length, beginning at which one can consider $T_W = T_\infty$ to a specified accuracy.

The ratio of the heat liberation coefficient from the cylinder surface to the value of heat liberation from a plane surface with constant thermal flux

$$\frac{Nu_x}{Nu_{xp}} = \frac{\theta_w(0)}{\theta_w(\xi)} \exp\left(-\frac{\xi^5}{\xi_m^5}\right)$$

is shown in Fig. 3. The competing effects of curvature and nonisothermality lead to a nonuniform increase, followed by a decrease in local heat liberation.

For a plane surface with exponentially decaying heat flux on the wall, a generalized solution can be obtained by using x/m as a longitudinal dimensionless coordinate. The equations for a plane wall were solved by the finite-difference method in [5]. The effect of curvature in these variables is shown in Fig. 4, where the case of a plane surface corresponds to $\xi_m = 0$. With the aid of Fig. 4 the height of the effective heat-liberating surface may be expressed as a fraction of m.

We will use approximate methods to obtain explicit calculation formulas. The most developed of these methods in the recent literature is that of the thin layer with boundary conditions of the first sort [4]. In specifying the thermal flux on the surface, after transformations we obtain [2]

$$Nu_{xp} = C^* Ra_x^{*1/5} \left(\frac{q_w x}{\int_0^x q_w dx} \right)^{1/5}, \tag{10}$$

$$C^* = \mathrm{Pr}/(4 + 4\,\mathrm{Pr}^{1/2} + 10\,\mathrm{Pr}). \quad (10)$$

For an exponentially decreasing heat flux on a lamina

$$\mathrm{Nu}_{xp} = C^*\,\mathrm{Ra}_x^{*1/5}\,\frac{\xi}{\xi_m}\left[\exp\left(\frac{\xi}{\xi_m}\right)^5 - 1\right]. \quad (11)$$

The effect of curvature can be considered with the aid of a Langmuir thermal layer

$$\mathrm{Nu}_x = \frac{x/r}{\ln\left(1 + \frac{1}{\mathrm{Nu}_{xp}}\cdot\frac{x}{r}\right)}. \quad (12)$$

Calculations with the approximate expressions (11), (12) for Pr = 0.7 are shown by the dashed lines of Fig. 3. The error increases with increase in longitudinal coordinate ξ. Equation (12) may be refined by consideration of an external "thick" layer [4].

NOTATION

x, r	are the longitudinal and radial coordinates;
u, v	are the projections of the velocity on axes x and r;
T	is the temperature;
q	is the thermal flux;
$\vartheta = (T - T_\infty)$	is the excess temperature;
ψ	is the flow function;
g	is the acceleration of gravity;
β	is the coefficient of volume expansion;
R	is the cylinder radius;
ν	is the kinematic viscosity coefficient;
a	is the thermal diffusivity;
λ	is the thermal conductivity;
f, θ	are the dimensionless flow function and temperature;
ξ, η	are the self-similar variables;
$\mathrm{Pr} = \nu/a$	is the Prandtl number;
$\mathrm{Gr}_x^* = g\beta q_0 x^4/\lambda\nu^2$, $\mathrm{Gr}_{xw}^* = g\beta q_w x^4/\lambda\nu^2$	are the modified Grashof numbers;
$\mathrm{Ra}_x^* = \mathrm{Gr}_x^*\mathrm{Pr}$	is the modified Rayleigh number;
$\mathrm{Nu}_x = \alpha_x x/\lambda$	is the Nusselt number.

Indices

w	is the wall;
0	is the initial value;
∞	is the surrounding medium;
p	is the plane plate;
x	is the local value.

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